TD6 Conduction thermique à symétrie cylindrique et effet Joule

IV

(1)

Correction Examen Janvier 2011
Thermodynamique statistique

(I) A) 1) $P_J = \sigma_U \times V$ = puissance dissipée par effet Joule

$$\sigma_U = \frac{P_J}{V} = \frac{RI^2}{V} = \frac{\rho_c L}{S_1} \times \frac{I^2}{S_1 L} = \frac{\rho_c I^2}{\pi^2 b_1^4}$$

$\sigma_U = 15\,502 \ W\,m^{-3}$

2) $0 \leq r \leq b_1$ Loi de Fourier $\vec{J_U} = -\lambda_c \frac{dT}{dr} \vec{e_r}$

(J_U indépendant de ϕ et z)

3) $\frac{\partial(\rho u)}{\partial t} = -\mathrm{div}\,\vec{J_U} + \sigma_u = 0$ (régime stationnaire)

$-\mathrm{div}(-\lambda_c \vec{\mathrm{grad}}\,T) + \sigma_u = 0$; $\lambda_c \Delta T + \sigma_U = 0$

$$\Delta T = \left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right\} = -\frac{\sigma_U}{\lambda_c}$$

$$\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) = -\frac{r\sigma_U}{\lambda_c} \Rightarrow r\frac{dT}{dr} = \frac{-\sigma_U}{\lambda_c}\frac{r^2}{2} + A$$

$$\frac{dT}{dr} = -\frac{\sigma_U}{2\lambda_c} r + \frac{A}{r} \quad \text{avec} \begin{cases} A = 0 \\ \text{sinon } \frac{dT}{dr} \to \infty \text{ en } r = 0 \end{cases}$$

$$T(r) = -\frac{\sigma_U}{4\lambda_c} r^2 + T(r=0) = \frac{-\rho_c I^2}{4\pi^2 b_1^4 \lambda_c} r^2 + T_0$$

$0 \leq r \leq b_1$

$$T(r=b_1) = T_c = \underbrace{\frac{-\rho_c I^2}{4\pi^2 b_1^2 \lambda_c}}_{1\,10^{-3} \text{ négligeable}} + T_0 = T_0$$

$\frac{dT}{dr}$ ds le conducteur négligeable car conducteur thermique parfait ($390 \ W\,m^{-1}\,K^{-1}$)

4) $I'_U(r) = \iint_{S_2} \vec{J}'_U \cdot \vec{n}\, dS = J'_U\, 2\pi r L$

avec $b_1 \leq r \leq b_2$ $\qquad J'_U(r) = \dfrac{I'_U(r)}{2\pi r L} = -\lambda_i \dfrac{dT}{dr}$

$$\underset{(0 \leq r \leq b_1)}{I_U(r)} = \underset{(b_1 \leq r \leq b_2)}{I'_U(r)} = P_J = \frac{\rho_c I^2}{\pi b_1^2} L$$

~~continu~~

$$\frac{dT}{dr} = -\frac{\rho_c I^2}{\pi b_1^2} \frac{L}{\lambda_i 2\pi r L} = \frac{-\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \frac{1}{r}$$

$$T(r) = \frac{-\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \ln r + B$$

$$T(r = b_2) = T_a = -\frac{\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \ln b_2 + B$$

$$B = T_a + \frac{\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \ln b_2$$

$$\boxed{T(r) = T_a + \frac{\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \ln\left(\frac{b_2}{r}\right)}$$

5) $T(r = b_1) = T_a + \dfrac{\rho_c I^2}{2\pi^2 b_1^2 \lambda_i} \ln\left(\dfrac{b_2}{b_1}\right) = T_c$

$$\lambda_i = \frac{\rho_c I^2}{2\pi^2 b_1^2 (T_c - T_a)} \ln\left(\frac{b_2}{b_1}\right) = 0{,}1\ \mathrm{W\,m^{-1}K^{-1}}$$

B

~~6) $P_J = \sigma_U \times V = \sigma_U \times \pi b_1^2 \times L$~~

~~↓ transmise par rayonnement de la surface latérale du fil à la surface latérale du conducteur extérieur~~